Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hours 30 minutes

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

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1. (a) Using the definitions of hyperbolic functions in terms of exponentials, show that

$$1 - \tanh^2 x \equiv \operatorname{sech}^2 x$$

(3)

(b) Solve the equation

$$2\operatorname{sech}^2 x + 3\tanh x = 3$$

giving your answer as an exact logarithm.

(3)

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Question 1 continued

(Total 6 marks)

Q1

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2. A curve has equation

$$y = \sqrt{9 - x^2} \qquad 0 \leqslant x \leqslant 3$$

(a) Using calculus, show that the length of the curve is $\dfrac{3\pi}{2}$

(4)

The curve is rotated through 2π radians about the x-axis.

(b) Using calculus, find the exact area of the surface generated.

(3)

Question 2 continued

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Question 2 continued

(Total 7 marks)

Q2

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3. $$\mathbf{M} = \begin{pmatrix} 3 & 1 & p \\ 1 & 1 & 2 \\ -1 & p & 2 \end{pmatrix} \quad \text{where } p \text{ is a real constant}$$

(a) Find the exact values of p for which $\mathbf{M}$ has no inverse.

(4)

Given that $\mathbf{M}$ does have an inverse,

(b) find $\mathbf{M}^{-1}$ in terms of p.

(5)

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Question 3 continued

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Question 3 continued

Q3

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4. (i)
$$f(x) = x \arccos x \qquad -1 \leqslant x \leqslant 1$$

Find the exact value of $f'(0.5)$.

(3)

(ii)
$$g(x) = \arctan(e^{2x})$$

Show that

$$g''(x) = k \operatorname{sech}(2x) \tanh(2x)$$

where k is a constant to be found.

(5)

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Question 4 continued

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Question 4 continued

Q4

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5. $$I_n = \int \sec^n x \, dx \qquad n \geqslant 0$$

(a) Prove that for $n \geqslant 2$

$$(n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$$

(6)

(b) Hence, showing each step of your working, find the exact value of

$$\int_0^{\frac{\pi}{4}} \sec^6 x \, dx$$

(4)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

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6. The line l_1 has equation

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k})$$

and the line l_2 has equation

$$\mathrm{r} = 2\mathbf{i} + s\mathbf{j} + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

where s is a constant and λ and μ are scalar parameters.

Given that l_1 and l_2 both lie in a common plane Π_1

(a) show that an equation for Π_1 is $3x + y - z = 3$

(4)

(b) find the value of s.

(1)

The plane Π_2 has equation $\mathbf{r}.(\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 3$

(c) Find an equation for the line of intersection of Π_1 and Π_2

(4)

(d) Find the acute angle between Π_1 and Π_2 giving your answer in degrees to 3 significant figures.

(4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 13 marks)

7. Using calculus, find the exact values of

(i) $\displaystyle\int_1^2 \frac{1}{x^2 - 4x + 5}\,\mathrm{d}x$

(3)

(ii) $\displaystyle\int_{\sqrt{3}}^3 \frac{\sqrt{x^2 - 3}}{x^2}\,\mathrm{d}x$

(5)

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Question 7 continued

Question 7 continued

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Q7

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8. The hyperbola H has equation

$$4x^2 - y^2 = 4$$

(a) Write down the equations of the asymptotes of H.

(1)

(b) Find the coordinates of the foci of H.

(2)

The point $P(\sec\theta, 2\tan\theta)$ lies on H.

(c) Using calculus, show that the equation of the tangent to H at the point P is

$$y\tan\theta = 2x\sec\theta - 2$$

(4)

The point $V(-1, 0)$ and the point $W(1, 0)$ both lie on H.

The point $Q(\sec\theta, -2\tan\theta)$ also lies on H.

Given that P, Q, V and W are distinct points on H and that the lines VP and WQ intersect at the point S,

(d) show that, as θ varies, S lies on an ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a and b are integers to be found.

(7)

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Question 8 continued

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Question 8 continued

(Total 14 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

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